

Application Note 05 – Accuracy, Resolution, and Repeatability



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1. Introduction

The **Electric Encoder™** includes fine and coarse channels whose output sine/cosine signals are digitized and processed to provide a digital code that represents the absolute position. The fine channel may have from 8 to 128 EC/Rs (Electrical Cycles/Revolution), depending on the diameter, and determines the encoder accuracy and resolution. The coarse channel has one, three, or seven EC/Rs, is used solely for the absolute position determination and does not affect the accuracy.

2. Precision and accuracy

Figure 1 illustrates the terms accuracy, resolution, and repeatability. The orange squares designate measured values; the X-axis designates a variable parameter, such as time. An ideal sensor should repeatedly provide the same output (or digital code), corresponding to the true position – shown by the black line. **Repeatability** is the extent to which, the same output value, whether correct or not, is repeatedly obtained for the same measured magnitude in the presence of an outside variable - all other variables being constant, it is often referred to as **precision**. In the case where the variable is time repeatability depends on electrical noise which is inherent in any physical measurement. This noise results from the discrete nature of the electric charge; it has a Gaussian amplitude distribution, constant spectral density, and zero average value. **Accuracy** which depends on the difference between the time-averaged measured value and the true value is depicted by the dashed black line.

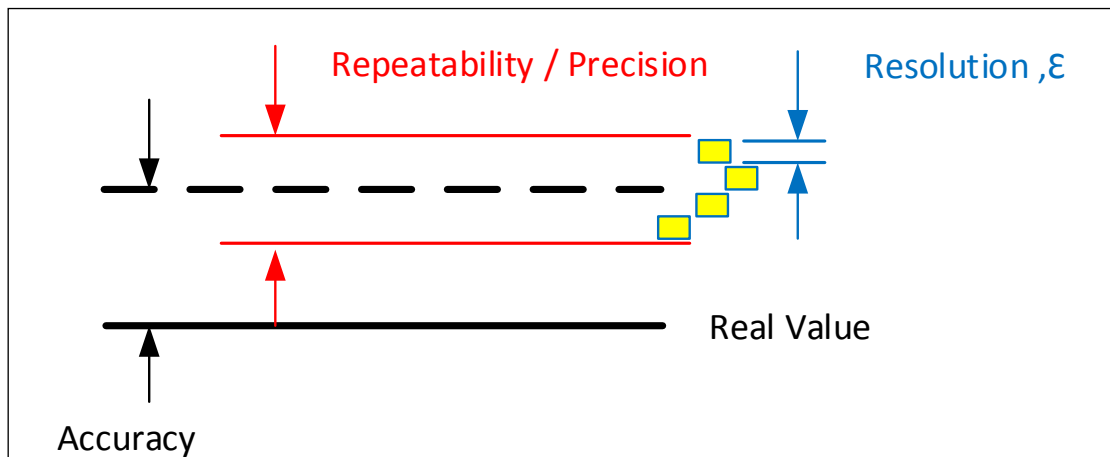


Figure 1

Resolution – shown by the small rectangles, is simply the minimal change in the measured value which still result in a discernible output change, it is usually smaller than the accuracy.

The output code of the encoder may also be affected by its history through physical memory mechanisms –see **Figure 10**; for example, thermal expansion/contraction may be partially irreversible due to mechanical creep moisture absorption etc, and the encoder may provide different codes at the same temperature and at the same position

Figure 2 illustrates **accuracy** and **precision** (repeatability) by means of probability distributions of successive samples of an encoder output with a fixed input **X** in the presence of electronic noise. The Y value reflects the probability of obtaining a true output value. The blue graph represents excellent **accuracy** (zero average error) but low **precision** (large variance due to large noise), the green and red graphs represent the same accuracy with successively less noise and thus higher precision. The purple graph represents medium accuracy and a precision.

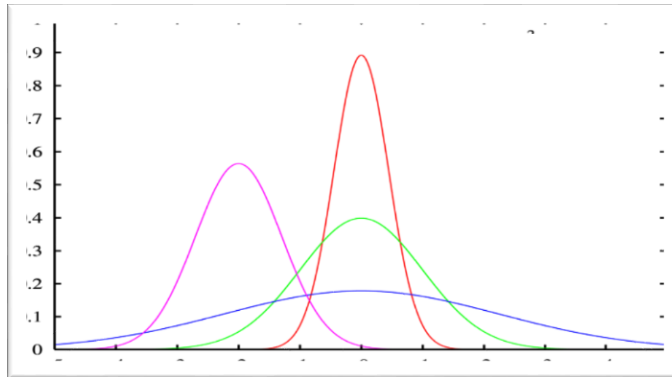


Figure 2

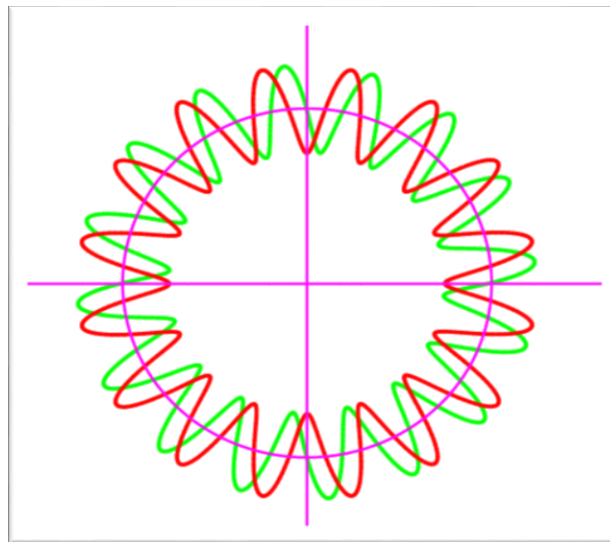


Figure 3

Since repeatability also sets a limit to the resolution and also cannot be mapped and compensated it is ultimately the defining factor and is synonymous with precision. **Figure 10** is a general model that includes the non-idealities which result in the encoder output error. The actual encoder is depicted by the red dashed box and its “ideal” portion in the blue box.

3. Angular resolution

Figure 3 illustrates graphically the output signals of the Fine-channel in a rotary **Electric Encoder™** with 16 electrical cycles per revolution (EC/R). The angular resolution of the encoder depends on the number **n** of EC/Rs in the Fine-channel and the number **N** of angular slices to which the electric cycle

can be divided; **N** depends on the number **M** of quantization steps in the A/D converter. Altogether, the analog input range of the A/D converter equals **Mxh** - where **h** is the quantization step – the maximum number of which is eventually limited by the inherent noise (-see below).

Figure 4-a illustrates the analog signals in polar coordinates (blue circle) over one electric cycle; each angular position has an uncertainty defined by the signal-to-noise ratio (blurred circle radius divided by **R**). In reality the quantized values of the sine and cosine signals lie on a virtual grid of spacing **h**. The grid includes all possible quantized values but in fact the allowed quantized pairs (blue dots) are further restricted to the vicinity of the circle as in **Figure 4-b** due to the interdependence of the Sine and Cosine where the radius vector to each dot determines its corresponding quantized angle.

If the input range of the A/D converter is represented by the diameter of the circle and equals **2R** then the number of quantization levels **M** equals **2R/h**. Unlike the amplitude quantized values the possible quantized angular increments which lie on the perimeter of the circle are not evenly spaced. For example, in region A the spacing $\Delta\theta$ between the quantized angles reaches a maximum value of h/R while in region B both larger and smaller values are encountered, in region C the maximum spacing would equal $0.5\sqrt{2}h/R$. Note, however that if, by rare chance, the circle exactly crosses the center point of the square C both digital words will increment simultaneously and the angular jump will amount to $\sqrt{2}h/R$ i.e., larger than in region A.

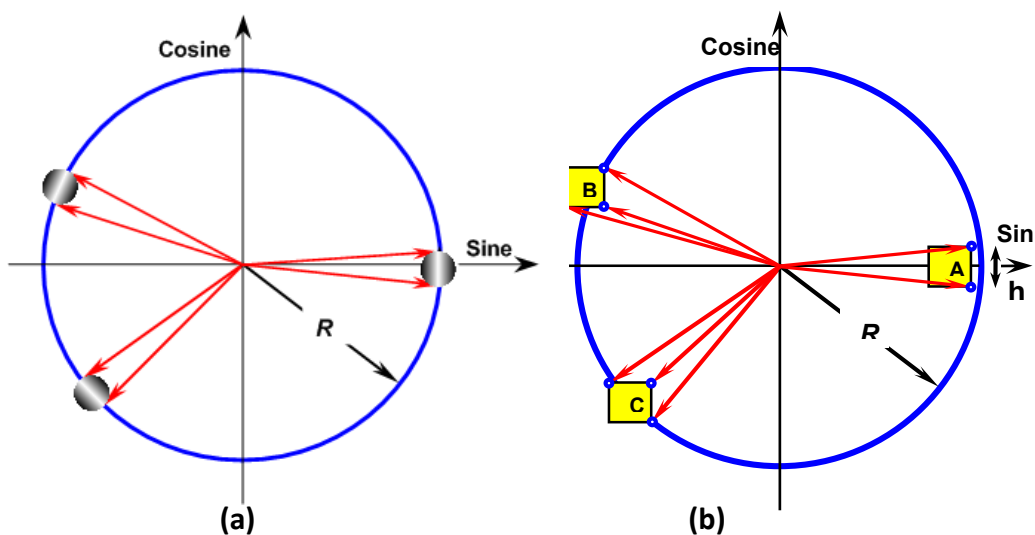


Figure 4

This is a largely of a theoretical and we will assume an average angular increment $\Delta\theta$ that equals h/R . This yields a total of **$N=2\pi R/h$** angular increments, i.e., **$N=\pi M$** . In practice, to allow for in the Sine and Cosine amplitude uncertainty the input range of the A/D converter should be larger than the signal peak-to-peak value, therefore, more conservatively, the number of angular increments in one electrical cycle can be assumed to be only 2 times as much as **M** – the number of signal amplitude levels i.e. **$N=2M$** . Taking into account the number **n** of EC/R the binary angular resolution would be **$\log_2 M + \log_2 n + 1$** bits.

The inherent noise density of the **Electric Encoder™** is fixed by design; it is nearly flat up to the encoder cutoff frequency **f₀** and its peak-to-peak value is proportional to $\sqrt{f_0}$. In applications where the full

signal bandwidth not required the noise can be reduced by further low pass filtering or by averaging (- see below). Thus reducing the cutoff frequency from its typical value of 1000Hz to 250Hz will improve a typical signal-to-noise ratio of 4000 to 8000 by a factor of 2 - equivalent to one extra bit. A more flexible alternative to bandwidth limiting is digital averaging, which provides further advantages, such as increasing the effective resolution of the A/D converter, and reducing the look up table size, as described below.

4. Over-sampling

Figure 5 illustrates the spread of an A/D converter output codes corresponding to a constant input due to its internal noise. The average of the scattered output codes converges at an intermediate value corresponding to the input value which is not constrained to the original grid. In other words, if the input signal has a maximum frequency f_0 and is sampled at a rate of $2kf_0$ then averaging each k successive samples would still satisfy the Nyquist sampling criterion while at the same time providing higher resolution. This Over sampling can be explained intuitively as follows:

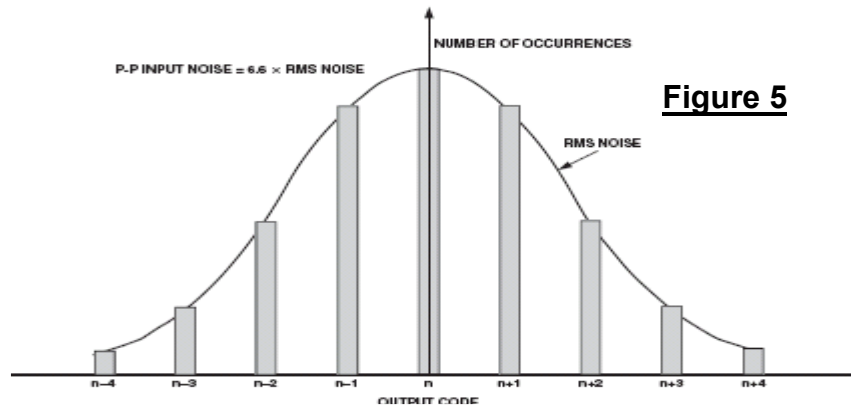


Figure 5

Thus, increasing the number k would decrease the quantization noise by a factor of \sqrt{k} and increase the resolution by $\log_2 \sqrt{k}$ bits.

The output code stream can be regarded as a sampled signal whose bandwidth is proportional to the sample rate. On the other hand the variance on the samples values is equivalent to a **constant amplitude** white noise of the same bandwidth. Therefore averaging – which is equivalent to low-pass filtering would decrease its amplitude in proportion to the square root of the bandwidth decrease. Thus, increasing the number k would decrease the quantization noise by a factor of \sqrt{k} and increase the resolution by $\log_2 \sqrt{k}$ bits.

Over-sampling particularly suits the **Electric Encoder™** with its typical 1kHz bandwidth since A/D converters can easily sample at least 100 times the required Nyquist rate thus improving quantization noise by a factor of $\sqrt{100}=10$. This would be equivalent to increasing the A/D converter resolution by $\log_2 10$ i.e., 3 extra bits e.g. converting a 12-bit A/D converter into a 15-bit converter.

It is important to notice that from the standpoint of the A/D converter the encoder noise is indistinguishable from its signal and the above analysis does not apply to it, it can only be reduced by limiting the signal bandwidth. In applications where the full bandwidth of the encoder is not needed this bandwidth reduction may be implemented by either analog low pass filtering of the input signals or by increasingly averaging the A/D converter output samples.

Also, instead of averaging the A/D converter output codes it is preferable to average the decoded angle obtained from the look-up table. In fact this provides two potential advantages: 1. Reducing the loading on the CPU. 2. The required look-up table stays the same and fit the A/D converter output codes rather than the enhanced resolution obtained by averaging.

5. Electric Encoder errors – general

The error components of the **Electric Encoder™** can be characterized by **revolution** errors and **cycle** error. Revolution errors result from imperfect rotational symmetry of the stator and rotor relative to the mechanical rotation axis, such as eccentricity, warp, or tilt. Cycle errors are due to imperfections in the Sine and Cosine signals and comprise harmonics of the EC (electric cycle): 1st harmonic error, 2nd and the 4th harmonic errors.

The number of **1st harmonic** cycles in one revolution is the same as the number of the encoder EC/R's and is a result of the sine and cosine offset voltages. The dashed circle in **Figure 6-a** belong to the "ideal" signals with no offset, while the blue circle represents the actual measured sine and cosine of the electrical angle with the red phasor represents the offset components. It is evident that at the two electrical angles where the two phasors are nearly collinear the decoded angle will not be affected by the offset while at the electrical angles where the two are nearly perpendicular the error will reach its maximum value of $\frac{o}{P}$ radians, where o is the offset phasor length and P is the peak amplitude of the encoder outputs. Thus an offset voltage of 1mV and a peak signal of 500mV will result in an electrical angle error of $1/500=2\text{mRad}$, which with 32 EC/R will translate into a mechanical error of $2/32\text{mRad}$.

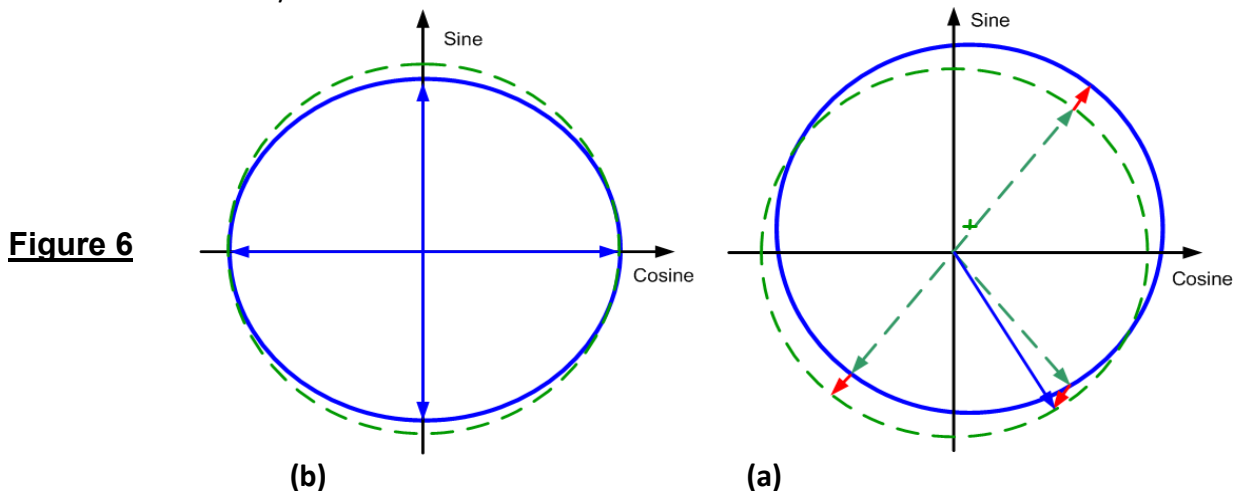


Figure 7-a illustrates the error graph of the DS-58 with 32 EC/R without offset compensation, **Figure 7-b** shows the residual error after offset compensation which comprises the revolution error and the **4th harmonic** error - which repeats four times per EC superimposed on it. This error is related to the physics of the encoder and can be compensated by introducing a 3rd order Taylor term in the gain curve of the sine and cosine. Otherwise it depends on the encoder type and is repeatable from unit to unit.

The corresponding offset and Taylor compensated error graph is shown in **Figure 7-c** where the revolution error, but no 2nd harmonic cycle error is noticeable. A **2nd harmonic** error could result from unequal amplitudes of the digitized sine and cosine signals, as in **Figure 6-b**, and from their imperfect quadrature relation. However, it is avoided by preserving the inherent near ideal amplitude matching of the **Electric Encoder™**, e.g. by using highly matched gain resistors (typically 0.01%) in any following gain stage prior to digitization or by software gain compensation applied to the digitized signals.

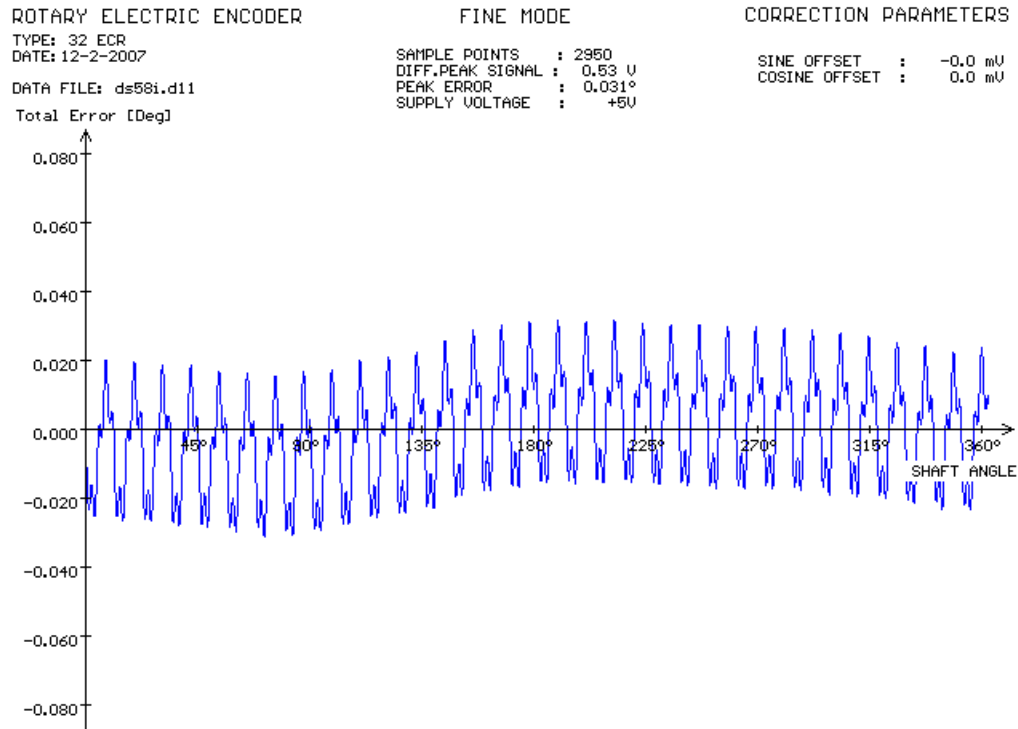


Figure 7-a

Figure 7-b

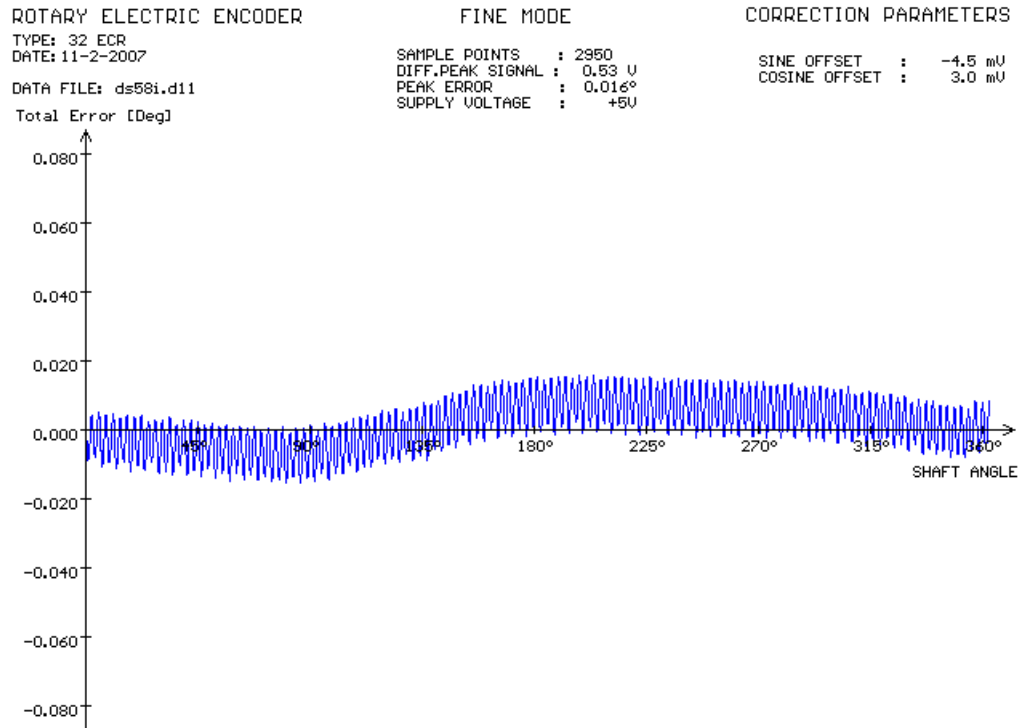
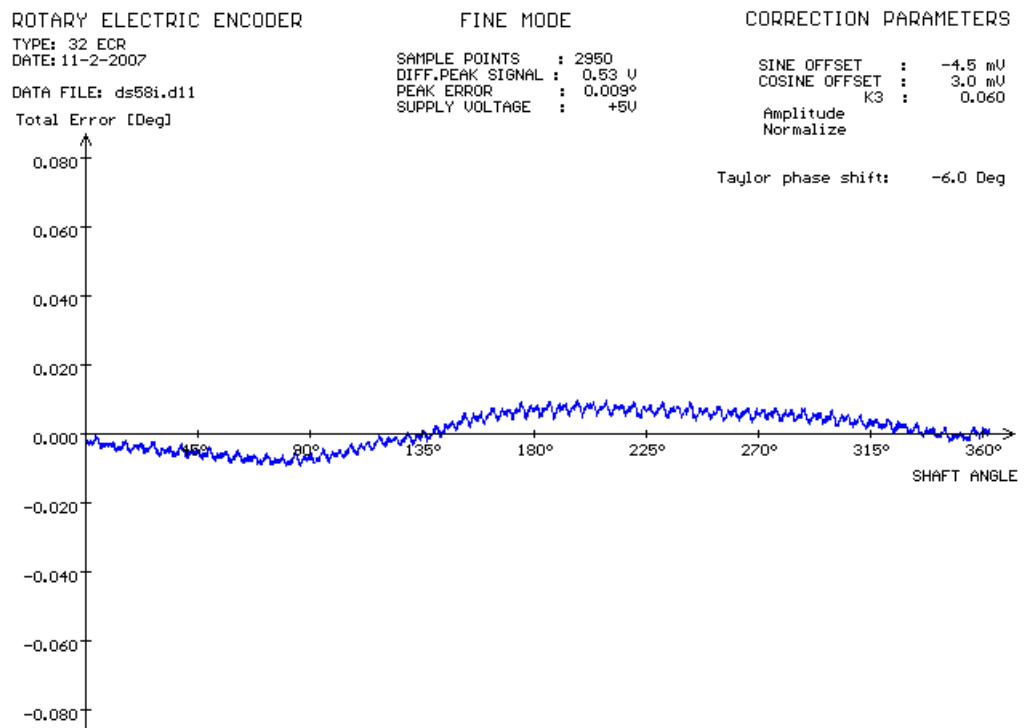


Figure 7-c



6. Temperature dependence

The stability of the **Electric Encoder™** output over temperature is a result of its holistic nature, which makes it nearly unaffected by thermal expansion, and on the common electronic channel for processing both Sine and Cosine. Residual temperature effects are partly due to slight output offset variations – see **Figure 8**.

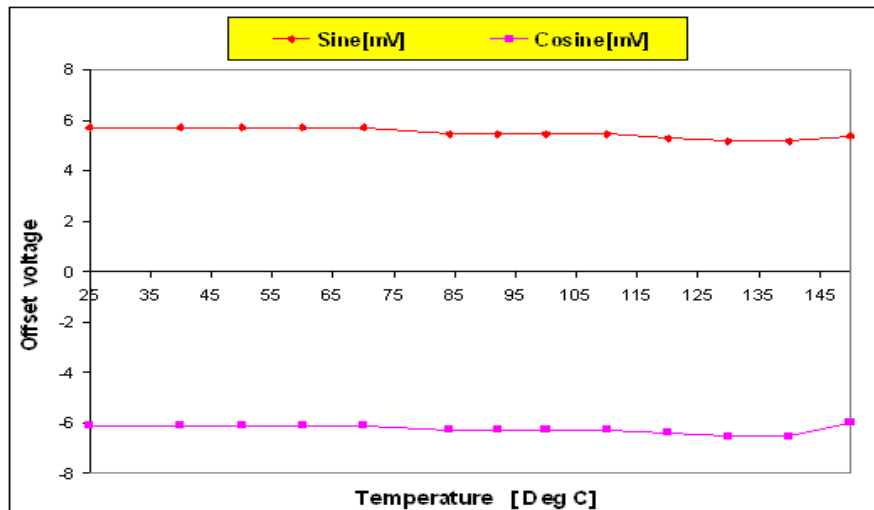


Figure 8

Figure 9 illustrates a typical output angle variation vs. temperature of a premium **DS-58** encoder with its rotor and stator clamped to a base plate to which the temperature sensor is attached as well. A close look reveals some irregularities around 35°C, which are due to the relatively fast temperature cycling which results in water condensing inside the encoder, as its cold inside lags the hotter, relatively vapor rich, ambient when going from low to high temperatures. An even faster temperature cycling, or larger thermal inertia of the mounting plate, could result in an apparent hysteresis (a further separation of the forward and backward curves) since the encoder and the temperature sensor may not be at the same temperature, depending on the location of the temperature sensor and the temperature slew rate.

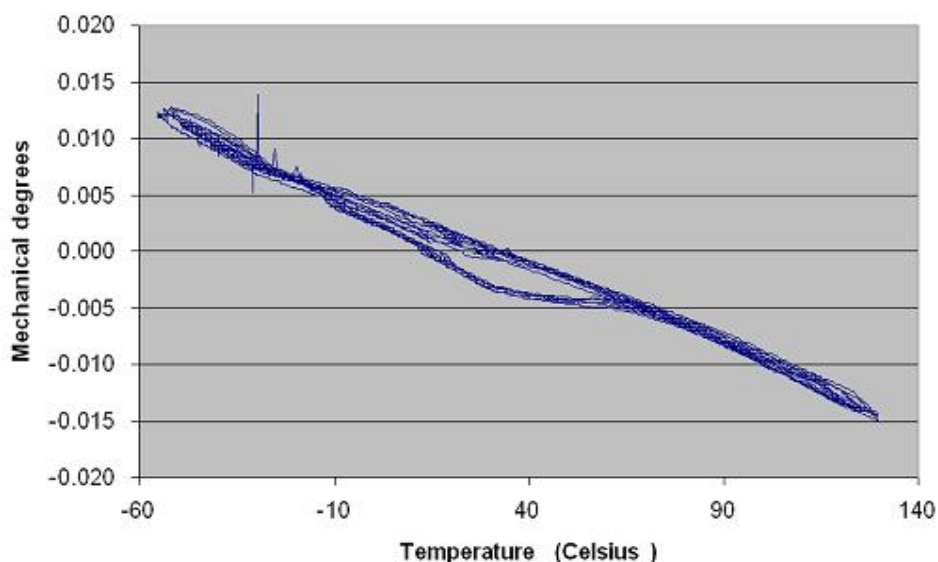


Figure 9

7. Phase behavior

Since capacitance can only be measured using time-varying voltages the raw signal in the **Electric Encoder™** is an amplitude-modulated carrier, which is internally processed by two synchronous-demodulators followed by low-pass filters to provide DC sine and cosine outputs. The cutoff frequency is determined by the carrier frequency and is highly flexible, it is limited by the internally generated noise, which sets a limit to the resolution, and is proportional to the square root of the cutoff frequency. In the standard DS encoders the cutoff frequency is 1kHz - sufficient for most applications.

Figure 10 illustrates the gain (red) and phase angle (blue) of the standard 1 kHz filter on a *logarithmic* frequency scale. **Figure 11** shows the nearly linear dependence of the phase inside the pass band on a *linear* frequency scale. The encoder output frequency which equals the rotation speed (in revolutions/second) times the number of EC/Rs determines the electrical phase angle of the output signals, but the *mechanical* phase angle equals the electrical phase angle *divided* by the number of EC/Rs. It follows that the mechanical phase angle is practically dependent on the rotation speed and on low pass filter regardless of the number of EC/Rs.

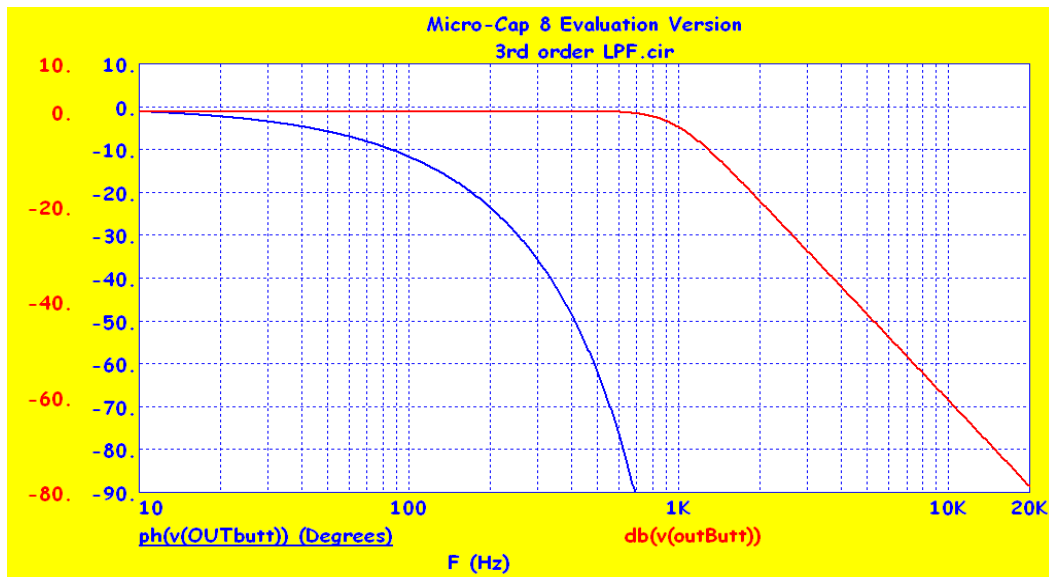
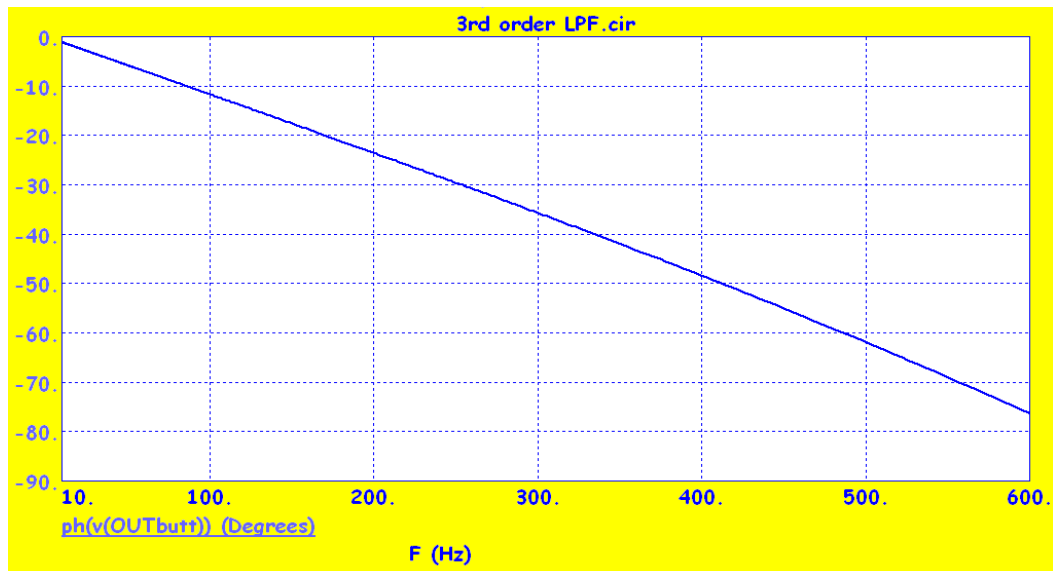


Figure 10

In other words, if the numbers on the frequency axis can also be interpreted as the number of shaft revolutions-per-second. For example, the phase shift is -24° at a signal frequency of 200 Hz; it is

also -24° mechanical degrees at 200 revolutions per second - irrespective of the number of EC/Rs. This is valid where the curve is linear which is, by definition, the useful frequency range.

Figure 11

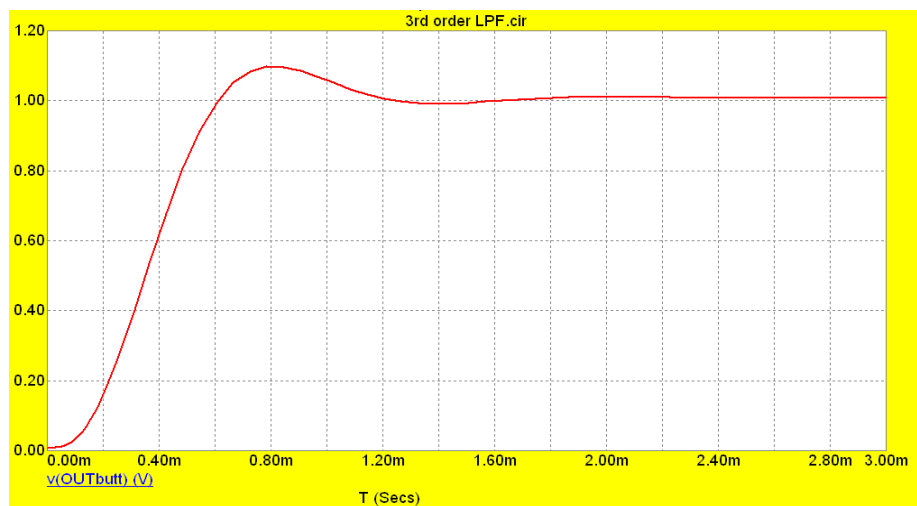


8. Latency and dynamic error

An ideal linear-phase is equivalent to a pure time delay, or latency; in the **Electric Encoder™** the phase is linear only over a portion of the frequency range, for this reason the respective step response shown in **Figure 12** is distorted as evidenced by the overshoot, besides rejection of the high frequency components. The practical result is a dynamic error which is proportional to the product of the angular speed and the latency. which is about for a 1 kHz filter.

Although the dynamic error of 600 µSec may result in a dynamic error which is significantly above the static error it is irrelevant except in rare applications, since other time constants such as motor and inertia dominate the dynamic loop behavior.

Figure 12



9. General error model

Figure 13 shows the various factors that affect the accuracy of the electric encoder (and any sensor, in general). The dashed blue box represents the “ideal” encoder while the red dashed box represents the actual encoder. $f_1(\theta)$ represents all deterministic errors i.e., errors that can be calibrated out, either by offset compensation, Taylor correction, or eventually mapping. Most of the temperature induced error is deterministic and can in principle be compensated if the encoder temperature is known, indirect temperature effects such as water condensation are non-deterministic. Similarly, electronic noise is non deterministic and can only be reduced by restricting the signal bandwidth.

The dynamic error results from the inherent signal delay and is proportional to the speed as such it can be compensated to some degree.

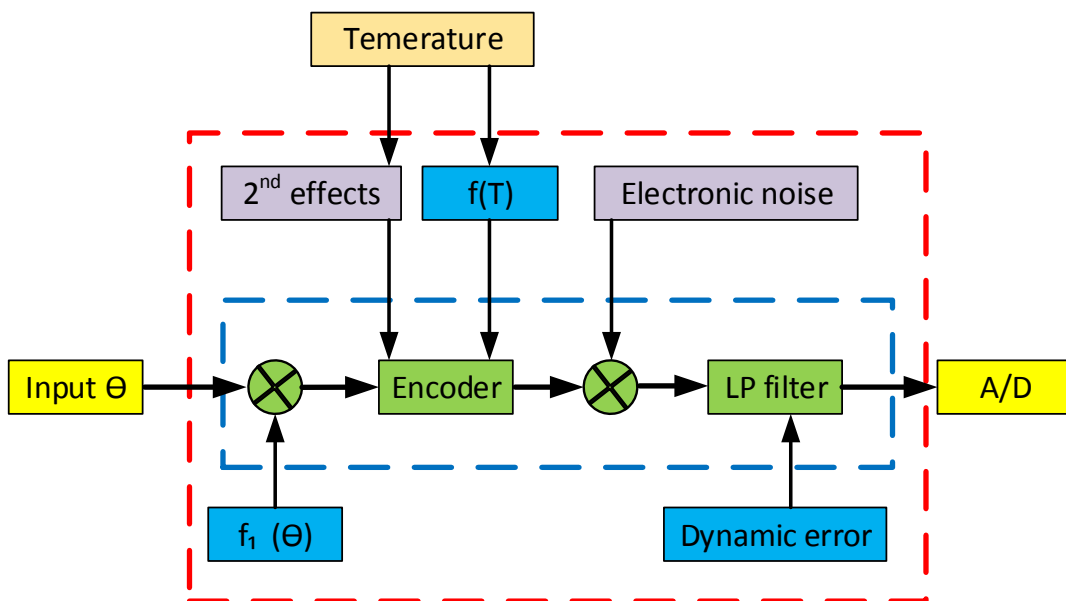


Figure 13